Resonance of Isochronous Oscillators

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Abstract An oscillator such that all motions have the same minimal period is called isochronous. When the isochronous is forced by a time-dependent perturbation with the same natural frequency as the oscillator the phenomenon of resonance can appear. This fact is well understood for the harmonic oscillator and we extend it to the nonlinear scenario.

1 Introduction

In this communication, we present some results from [4] that aim to characterize the class of periodic forcings producing resonances in nonlinear isochronous oscillators.

A well-known fact from physics and mathematics is that the harmonic oscillator with period 2π perturbed by a periodic forcing

$$\ddot{x} + n^2 x = p(t),$$

$n = 1, 2, \ldots$, exhibits resonance whenever the Fourier coefficient

$$\hat{p}_n := \frac{1}{2\pi} \int_0^{2\pi} p(t) e^{-int} dt$$

does not vanish. In this context, resonance means that all solutions of the perturbed equation are unbounded. After this example, the question that naturally arises is if there exists an equivalent condition for general nonlinear isochronous oscillators. As far as we know, this question was first raised by Prof. Roussarie in the Open Problems Session of the II Symposium on Planar Vector Fields (Lleida, 2000).

In this direction, Ortega [3] proved that if the nonlinear isochronous oscillator satisfies a Lipschitz condition then there exist functions $p(t)$ producing resonance.

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