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Periodic oscillators, isochronous centers and resonance

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Abstract

An oscillator is called isochronous if all motions have a common period. When the system is forced by a time-dependent perturbation with the same period the dynamics may change and the phenomenon of resonance can appear. In this context, resonance means that all solutions are unbounded. The theory of resonance is well known for the harmonic oscillator and we extend it to nonlinear isochronous oscillators.

Keywords: isochronous center, oscillator, resonance, perturbation
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1. Introduction

Consider an oscillator with equation

$$\ddot{x} + V'(x) = 0, \; x \in \mathbb{R} \tag{1}$$

and assume that it has an isochronous center at the origin. This means that $x = 0$ is the only equilibrium of the equation and the remaining solutions are periodic with a fixed period, say $T = 2\pi$. We are interested in the phenomenon of resonance for periodic perturbations. More precisely, we ask for the class of 2π-periodic functions $p(t)$ such that all the solutions of the non-autonomous equation

$$\ddot{x} + V'(x) = \varepsilon p(t) \tag{2}$$

are unbounded. Here $\varepsilon \neq 0$ is a small parameter.

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