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Crossing Periodic Orbits via First Integrals

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We characterize the families of periodic orbits of two discontinuous piecewise differential systems in $\mathbb{R}^3$ separated by a plane using their first integrals. One of these discontinuous piecewise differential systems is formed by linear differential systems, and the other by nonlinear differential systems.

Keywords: Discontinuous piecewise differential systems; first integrals; periodic orbits; homoclinic orbits.

1. Introduction and Statement of the Main Results

Discontinuous piecewise differential systems have been studied intensively in recent decades and this is mainly due to the fact that they provide in many applications more realistic models, as for instance in modeling electrical circuits [di Bernardo *et al.*, 2008; Simpson, 2010], in some mechanical problems [Andronov *et al.*, 1966], in control theory problems [Barbashin, 1970], etc. We define a *discontinuous piecewise differential system* following the rules of Filippov [1988].

In the study of the dynamics of differential equations the periodic orbits play a main role when they exist, and the crossing periodic orbits play also a relevant role in the case of the discontinuous piecewise differential systems. A *crossing periodic orbit* of a discontinuous piecewise differential system in $\mathbb{R}^3$ is a periodic orbit which intersects the surface of discontinuity in a finite number n of points with $n > 1$.

The objective of this paper is to study the crossing periodic orbits of discontinuous piecewise differential systems of the form

$$X(x,y,z) = X_1(x,y,z) + \mathrm{sign}(x)X_2(x,y,z), \qquad (1)$$

where X_1 and X_2 are smooth vector fields in $\mathbb{R}^3$ using their first integrals. As usual, the sign function is defined as

$$\mathrm{sign}(x) = \begin{cases} -1 & \text{if } x < 0, \\ 0 & \text{if } x = 0, \\ 1 & \text{if } x > 0. \end{cases}$$

We note that the discontinuous piecewise differential system (1) is separated by the plane $x = 0$, and it is given by the system $Y = X_1 + X_2$ in $x > 0$, and by the system $Z = X_1 - X_2$ in $x < 0$. If H_1 and H_2 are first integrals of Y and Z respectively, then

$$H = \frac{H_1 + H_2}{2} + \mathrm{sign}(x)\left(\frac{H_1 - H_2}{2}\right),$$