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Periods of holomorphic maps on compact Riemann surfaces and product of spheres

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ABSTRACT
In this article, we consider non-constant holomorphic maps on Riemann surfaces and product of Riemann spheres, and we give conditions on the maps in order that they have arbitrary large prime numbers as periods. We use Lefschetz fixed point theory and in particular we compute the Lefschetz numbers of period *m* for large *m*'s.

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1. Introduction

In the theory of the dynamical systems and mainly in the study of the iteration of self-maps on a topological manifold X, the periodic orbits play an important role. More precisely, let $f: X \to X$ be a continuous map, a point $x \in X$ is *periodic* of *period* $k \in \mathbb{N}$ if $f^k(x) = x$ and $f^j(x) \neq x$ for $j = 1, \ldots, k-1$. The set $\{x, f(x), \ldots, f^{k-1}(x)\}$ is the *periodic orbit* of the periodic point x of period k. If $k = 1$, then the periodic point x is called a *fixed point*. We shall denote by $Per(f)$ the set of periods of the map f. A natural question is to ask for all the possible periods that the map f can exhibit. In some situations, the knowledge of some possible periods of the map gives understanding of some global properties of the dynamics of the map, as in the case for continuous self-maps on the interval. If a continuous map on the interval has a periodic orbit of period three, then the map has orbits of all possible periods (*cf.* [9, 12]).

The differential topological methods are very useful for understanding the periodic structure of continuous self-maps on manifolds on dimensions greater than 1, because the topology of the manifold plays an important role, in particular we use the Lefschetz fixed point theory.

In this article, we study the periodic structure of non-constant holomorphic maps on Riemann surfaces and product of Riemann spheres, in particular we give conditions on homology such that the maps have arbitrary large prime numbers as periods; the

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Dedicated to O.M. Sharkovsky in his 82nd birthday.