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Limit Cycles Bifurcating from a Family of Reversible Quadratic Centers via Averaging Theory

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Consider the class of reversible quadratic systems

$$\dot{x} = y, \quad \dot{y} = -x + x^2 + y^2 - r^2,$$

with $r > 0$. These quadratic polynomial differential systems have a center at the point $((1 - \sqrt{1+4r^2})/2, 0)$ and the circle $x^2 + y^2 = r^2$ is one of the periodic orbits surrounding this center. These systems can be written into the form

$$\dot{x} = y + (4 + A)x^2 - Ay^2, \quad \dot{y} = -x,$$

with $A \in (-2, 0)$. For all $A \in \mathbb{R}$ we prove that the averaging theory up to seventh order applied to this last system perturbed inside the whole class of quadratic polynomial differential systems can produce at most two limit cycles bifurcating from the periodic orbits surrounding the center $(0, 0)$ of that system. Up to now this result was only known for $A = -2$ (see [Li, 2002; Liu, 2012]).

Keywords: Limit cycle; quadratic reversible center; averaging theory.

1. Introduction and Statement of the Main Results

One of the important problems in the qualitative theory of differential equations is the study of their limit cycles. The second part of the well-known Hilbert's 16th problem [Smale, 1998] asks about the maximal number and the possible relative positions of limit cycles in the planar polynomial differential systems of degree $n \geq 2$. This problem is still open, even for the case $n = 2$.

An easier problem than the Hilbert's 16th problem is the study of the number of limit cycles which can bifurcate from the periodic orbits surrounding a center of a polynomial differential system. Many authors these last years have studied this last problem restricted to the centers of the quadratic polynomial differential systems, see for instance, the book by Christopher and Li [2007] and the hundreds of references quoted therein.

The tools for studying the limit cycles bifurcating from the periodic orbits surrounding a center are