Journal of Nonlinear Mathematical Physics, Vol. 27, No. 3 (2020) 414–428

On the global dynamics of a three-dimensional forced-damped differential system

Jaume Llibre
Departament de Matemàtiques, Universitat Autònoma de Barcelona,
08193 Bellaterra, Barcelona, Catalonia, Spain
jllibre@mat.uab.cat

Y. Paulina Martínez
Centre de Recerca Matemática, Campus Universitat Autònoma de Barcelona
08193 Bellaterra, Barcelona, Catalonia, Spain
Departamento de Matemática, Facultad de Ciencias, Universidad del Bío-Bío,
Casilla 5-C, Concepción, Chile
yohanna.martinez@uab.cat; ymartinez@ubiobio.cl

Claudia Valls
Departamento de Matemática, Instituto Superior Técnico, Universidade Técnica de Lisboa,
Av. Rovisco Pais 1049–001, Lisboa, Portugal
cvalls@math.ist.utl.pt

Received 30 April 2019

Accepted 14 October 2019

In this paper by using the Poincaré compactification of $\mathbb{R}^3$ we make a global analysis of the model $x' = -ax + y + yz$, $y' = x - ay + bxz$, $z' = cz - bxy$. In particular we give the complete description of its dynamics on the infinity sphere. For $a + c = 0$ or $b = 1$ this system has invariants. For these values of the parameters we provide the global phase portrait of the system in the Poincaré ball. We also describe the α and ω-limit sets of its orbits in the Poincaré ball.

Keywords: Global dynamics; Poincaré compactification; forced-damped system; invariant algebraic curve; invariant.

2000 Mathematics Subject Classification: 34C05, 34C07, 34C08

1. Introduction and statement of the main results

We consider the autonomous polynomial differential system

$$\begin{aligned} \dot{x} &= -ax + y + yz, \\ \dot{y} &= x - ay + bxz, \\ \dot{z} &= cz - bxy, \end{aligned} \tag{1.1}$$

where a, b, c are real parameters and $b > 0$. As usual the dot denotes derivative with respect to the time t. This system was proposed and studied by Pehlivan extending a previous study of Craik and Okamoto [2] including linear forcing and damping, for more details on that system and on the notions of forcing and damping see [9]. It is a relevant system because it arises in mechanical,