Journal of Mathematical Physics ARTICLE scitation.org/journal/jmp

On the periodic solutions of the relativistic driven harmonic oscillator

Cite as: J. Math. Phys. 61, 012501 (2020); doi: 10.1063/1.5129377
Submitted: 27 September 2019 • Accepted: 19 December 2019 •
Published Online: 10 January 2020

Jaume Llibre[1,a)] and Amar Makhlouf[2,b)]

AFFILIATIONS

[1] Departament de Matematiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain
[2] Department of Mathematics, Laboratory LMA, University of Annaba, Elhadjar, 23 Annaba, Algeria

[a)] jllibre@mat.uab.cat
[b)] makhloufamar@yahoo.fr

ABSTRACT

Using the averaging theory, we prove the existence of periodic orbits with small velocities with respect to the speed of light in the forced harmonic oscillator with relativistic effects in dimension one.

I. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

For the forced harmonic oscillator with relativistic effects in dimension one with the equation of motion

$$\left(\frac{mx'}{\sqrt{1-\frac{x'^2}{c^2}}}\right)' + kx = F_0\cos(\omega t), \tag{1}$$

we prove the existence of periodic orbits with small velocities with respect to the speed of light. Here, $c > 0$ is the speed of light in the vacuum, $m > 0$ is the mass at rest, $k > 0$ is the spring stiffness coefficient, and ω and F_0 are the frequency and amplitude of the external force [for more details on this equation, see Ref. 14].

These last years, some results have been published on the periodic solutions of different oscillators with relativistic effects, as the relativistic forced pendulum [see, for instance, the works of Brezis and Mawhin,[3] Bereanu and Torres,[1] and Maró].[13] These authors have obtained their results using variational and topological methods. The stability of the equilibrium $x = 0$ of the relativistic pendulum with variable length has been proved by Chu, Lei, and Zhang in Ref. 7.

Kim and Lee in Ref. 9 have studied numerically the existence of chaotic motion in the relativistic harmonic oscillator. Moreover, the existence of chaotic motion of relativistic particles has been studied in different contexts by several authors [see, for instance, Refs. 2, 5, 6, and 10].

Nuñez and Rivera in Ref. 14 followed the ideas of this previous work, and assuming that

(i) $\frac{k}{m} < \frac{\omega^2}{16}$,
(ii) $F_0 < \frac{1}{4}mc\omega$, and
(iii) $\frac{(mc\omega)^{19}((mc\omega)^2 - 16F_0^2)}{120\pi F_0^2((mc\omega)^2 + 4F_0^2)^{19/2}} \sin\left(\frac{6\pi(\omega k)^{1/2}mc^{3/2}}{((mc\omega)^2 + 4F_0^2)^{3/4}}\right) > 1$,

proved the existence of a periodic orbit for the relativistic driven harmonic oscillator (1) via lower and upper solutions.

Our result shows the existence of periodic orbits in the relativistic driven harmonic oscillator with relativistic effects (1), under the assumptions that