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Periodic Solutions of Continuous Third-Order Differential Equations with Piecewise Polynomial Nonlinearities

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We consider third-order autonomous continuous piecewise differential equations in the variable x. For such differential equations with nonlinearities of the form x^m, we investigate their periodic solutions using the averaging theory. We remark that since the differential system is only continuous we cannot apply to it the classical averaging theory, that needs that the differential system be at least of class C^2.

Keywords: Periodic solution; averaging method; third-order differential equation; piecewise differential equation.

1. Introduction and Statement of the Main Result

For studying some electrical circuits, Sprott [2000a, 2000b], and Sun and Sprott [2009] considered the third-order differential equation $\dddot{x} = -\dot{x} - a\ddot{x} + g(x)$, where g is an elemental piecewise function. The authors showed that some of these equations exhibit chaos.

In this paper, we are interested in studying the third-order differential equations of the form

$$\dddot{x} = -\dot{x} + \varepsilon|\ddot{x}| - \varepsilon a x^m, \qquad (1)$$

where a and ε are parameters and ε is small. But our interest is in studying how their periodic solutions depend on the parameter a and on the exponent m. Another interesting point is that the differential equation (1) is only continuous and nonsmooth, and there are very few tools for studying the periodic solutions of such differential equations analytically. Here we illustrate one such tool.

We can write the third-order differential equations (1) as the following differential system of first order

$$\dot{x} = y,$$