Topological Methods in Nonlinear Analysis
Volume 55, No. 2, 2020, 387–402
DOI: 10.12775/TMNA.2020.004

ON THE CENTERS OF CUBIC POLYNOMIAL DIFFERENTIAL SYSTEMS WITH FOUR INVARIANT STRAIGHT LINES

Jaume Llibre

Abstract. Assume that a cubic polynomial differential system in the plane has four invariant straight lines in generic position, i.e. they are not parallel and no more than two straight lines intersect in a point. Then such a differential system only can have 0, 1 or 3 centers.

1. Introduction and statement of the main results

A *center* of a differential system in $\mathbb{R}^2$ is an equilibrium point p for which there exists a neighbourhood U of p such that $U \setminus \{p\}$ is filled by periodic orbits. The equilibrium point p is a *focus* if there exists a neighbourhood U of p where all the orbits in $U \setminus \{p\}$ spiral tending to p either in backward, or in forward time. These definitions of focus and center goes back to Dulac [10] and Poincaré [23].

The problem of distinguish between a focus or a center (known as the *center-focus problem*) is a classical problem in the qualitative theory of planar polynomial differential systems, which is related to the Hilbert 16th problem, see Hilbert [14], Ilyashenko [15], Li [19].

2020 *Mathematics Subject Classification.* 37K10, 37C27, 37K05.

Key words and phrases. Cubic system; cubic polynomial differential systems; centers; invariant straight line.

The author is partially supported by the Ministerio de Ciencia, Innovación y Universidades, Agencia Estatal de Investigación grants MTM2016-77278-P (FEDER), the Agència de Gestió d'Ajuts Universitaris i de Recerca grant 2017SGR1617, and the H2020 European Research Council grant MSCA-RISE-2017-777911.