Weak Focus, Limit Cycles, and Bifurcations for Bounded Quadratic Systems

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We study the bounded quadratic systems with either two weak foci or a weak focus of order 2. From the first case we obtain $(1, 1)$-configuration of limit cycles, and in the second case we prove that there is no limit cycle surrounding the weak focus of order 2. Also, we unfold the bounded quadratic system with a center, and the maximum number of limit cycles in this case is two. © 1995 Academic Press, Inc.

1. INTRODUCTION

We consider the differential system $\dot{x} = dx/dt = P(x, y)$, $\dot{y} = dy/dt = Q(x, y)$, where P and Q are polynomials with real constant coefficients, and x, y, and t are also real. When the maximum of the degrees of P and Q is 2 we call such systems *quadratic systems*. If all the trajectories of a quadratic system remain bounded for $t \geq 0$ we say that it is *bounded*; we abbreviate *bounded quadratic system* as BQS.

In 1970 Dickson and Perko [DP] determined all possible phase portraits for BQS without taking into account whether or not limit cycles

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