STRUCTURAL STABILITY OF PLANAR HAMILTONIAN POLYNOMIAL VECTOR FIELDS

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ABSTRACT

We characterize the set of all structurally stable planar Hamiltonian polynomial vector fields with respect to perturbations, first in the set of all C^r planar vector fields, second in the set of all planar polynomial vector fields, and third in the set of all planar Hamiltonian polynomial vector fields. We also classify the canonical regions of the structurally stable planar Hamiltonian polynomial vector fields, and study its genericity.

1. *Introduction*

In 1990 Shafer **[16]** characterized the planar gradient polynomial vector fields which are structurally stable in two levels: in the set of all C^r planar vector fields and in the set of all planar polynomial vector fields. He also presented sufficient conditions for structural stability in the set of all planar gradient polynomial vector fields.

The first goal of this paper is to prove results equivalent to Shafer's but for planar Hamiltonian polynomial vector fields. More precisely, we characterize the planar Hamiltonian polynomial vector fields which are structurally stable with respect to perturbations, first in the set of all C^r planar vector fields (Theorem 4.2), second in the set of all planar polynomial vector fields (Theorem 4.4), and finally in the set of all planar Hamiltonian polynomial vector fields (Theorem 4.13).

When we consider only polynomial perturbations, we can consider the vector fields either on $\mathbb{R}^2$ or on $\mathbb{S}^2$ (Poincaré sphere) but we will prove that the set of all structurally stable planar Hamiltonian polynomial vector fields is the same in both cases.

We also consider the interesting question in the definition of structural stability of whether or not it is important that the equivalence homeomorphism between the structurally stable vector field and its neighbours be close to the identity map.

By removing all the separatrices in the phase portrait of a planar Hamiltonian polynomial vector field we get its canonical regions. We classify all canonical regions of the structurally stable planar Hamiltonian polynomial vector fields with respect to the three previous types of perturbations.

Finally, we show that, of the three sets of structurally stable planar Hamiltonian polynomial vector fields corresponding to the three types of perturbations, the only one which is generic is the set of structurally stable planar Hamiltonian polynomial vector fields with respect to perturbations in the set of all planar Hamiltonian polynomial vector fields (here, generic means that the set is open and dense in the set of all planar Hamiltonian polynomial vector fields).

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