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On Co-Orbital Quasi-Periodic Motion in the Three-Body Problem*

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Abstract. Within the framework of the planar three-body problem we establish the existence of quasi-periodic motions and KAM 4-tori related to the co-orbital motion of two small moons about a large planet where the moons move in nearly circular orbits with almost equal radii. The approach is based on a combination of normal form and symplectic reduction theories and the application of a KAM theorem for high-order degenerate systems. To accomplish our results we need to expand the Hamiltonian of the three-body problem as a perturbation of two uncoupled Kepler problems. This approximation is valid in the region of phase space where co-orbital solutions occur.

Key words. three-body problem, symplectic scaling, co-orbital regime, 1:1 mean-motion resonance, normalization and reduction, KAM theory for multiscale systems, quasi-periodic motion and invariant 4-tori

AMS subject classifications. 70F10, 70K65, 37J40, 37N05

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1. Introduction.

Saturn is surrounded by groups of rings and more than 50 moons. Two of these small moons, named Janus and Epimetheus, almost share the same orbit in their travel around Saturn. So, it seems that their different orbital speeds should make them crash into each other. From Kepler's laws, the inner moon has smaller period and then traps the outer moon. But due to their mutual gravitational attraction they never get closer than about 15,000 kilometers from each other. Instead of crashing, they exchange orbital positions once every four years. Figure 1 shows a schematic drawing of the path of the so-called co-orbital moons.

Celestial mechanics refers to co-orbital motion when two or more bodies, such as planets, moons, or asteroids, orbit at the same or very similar distance from their central body, that is, they are in a 1:1 mean-motion resonance. There are several types of co-orbital objects, depending on their point of libration. One class is the trojan, which librates around one of the two stable Lagrangian points, called L_4 and L_5, 60° ahead of and behind the central body, respectively. The most well-known examples are the asteroids that orbit ahead of or behind Jupiter around the Sun. Today, the research of trojan planets is beyond our solar system.

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