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Qualitative study of the hyperbolic collision restricted three-body problem*

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Abstract. We have two mass points of equal masses $m_1 = m_2 > 0$ moving under Newton's law of gravitational attraction in a collision hyperbolic orbit while their centre of mass is at rest. We consider a third mass point, of mass $m_3 = 0$, moving on the straight line L perpendicular to the line of motion of the first two mass points and passing through their centre of mass. Since $m_3 = 0$, the motion of the masses m_1 and m_2 is not affected by the third mass and from the symmetry of the motion it is clear that m_3 will remain on the line L. The hyperbolic collision rectricted three-body problem consists in describing the motion of m_3. Our main result is the characterization of the global flow of this problem.

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1. Introduction

The purpose of dynamics is to characterize all the possible qualitative motions of a given dynamical system. Such classification for the Newtonian three-body problem is still an open problem. This has led to the study of various simplifications or restrictions of it. This includes the restricted three-body problems in which one of the masses is assumed to be zero. We consider the case when the two positive masses are not periodically attracted and the infinitesimal mass point is moving on a line.

More specifically, let $m_1 = m_2 = 1$ be two mass points moving under Newton's law of gravitation in a hyperbolic collision orbit on the x-axis while their centre of mass is fixed at the origin of coordinates. We consider a third mass point with infinitesimal mass moving on the y-axis (see figure 1). As usual these two masses are called *primaries*. Since $m_3 = 0$ the motion of the first two mass points is not affected by the third and from the symmetry of the motion it is clear that the third mass point will remain on the y-axis. The problem is to study the motion of the infinitesimal mass, and then we have a restricted three-body problem that we call the *hyperbolic collision restricted problem*. The equations of motion of this problem in the phase space $(y, \dot{y}, t)$ are given in section 2. Every solution of the hyperbolic collision restricted problem are defined for all time t, i.e. from $-\infty$ to ∞, except for those that start or end at a triple collision.

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