

Limit Cycles for Non Smooth Differential Equations via Schwarzian Derivative

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We consider a class of planar polynomial systems with discontinuous righthand sides and prove that, under certain hypotheses, it presents at most one singular limit cycle and two regular limit cycles. Furthermore the sum of the multiplicity of the regular limit cycles is less or equal than two. A key point in the proof is the study of the Schwarzian derivative of the return map. Finally, we give some examples reaching these bounds. © 1996 Academic Press, Inc.

1. INTRODUCTION AND MAIN RESULTS

In this paper we study the maximum number of limit cycles of the differential equation with discontinuous righthand sides

$$(\dot{x}, \dot{y}) = (P(x, y), Q(x, y)) = \begin{cases} (P^+(x, y), Q^+(x, y)) & \text{if } y \geq 0, \\ (P^-(x, y), Q^-(x, y)) & \text{if } y \leq 0, \end{cases} \quad (1)$$

where (P, Q) will be given in (2). We denote the x -axis by $L = \{(x, y) \in \mathbb{R}^2 : y = 0\}$, and the open upper (resp. lower) half plane by Π^+ (resp. Π^-). Note that L is, possibly, a discontinuous line for system (1).

System (1) covers a wide range of physical phenomena, see for instance [AKV]. For these kind of differential equations the concept of solution needs to be re-examined and redefined in this setting. We will follow the

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