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Invariant fibrations for some birational maps of $\mathbb{C}^2$

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ABSTRACT

In this article, we extract and study the zero entropy subfamilies of a certain family of birational maps of the plane. We find these zero entropy mappings and give the invariant fibrations associated to them.

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1. Introduction

A mapping $f = (f_1, f_2) : \mathbb{C}^2 \longrightarrow \mathbb{C}^2$ is said to be rational if each coordinate function is rational, that is, f_i is a quotient of polynomials for $i = 1, 2$. These maps can be naturally extended to the projective plane $P\mathbb{C}^2$ by considering the embedding $(x_1, x_2) \in \mathbb{C}^2 \to [1 : x_1 : x_2] \in P\mathbb{C}^2$. The induced mapping $F : P\mathbb{C}^2 \longrightarrow P\mathbb{C}^2$ has three components $F_i[x_0 : x_1 : x_2]$ which are homogeneous polynomials of the same degree. If F_1, F_2, F_3 have no common factors and have degree d, we say that f or F has degree d. Similarly we can define the degree of $F^n = F \circ \cdots \circ F$ for each $n \in \mathbb{N}$.

We are interested in birational maps. It is said that a rational mapping $f : \mathbb{C}^2 \longrightarrow \mathbb{C}^2$ is birational if there exists an algebraic curve C and another rational map g such that $f \circ g = g \circ f = id$ in $\mathbb{C}^2 \setminus C$.

The study of the dynamics generated by birational mappings in the plane has been growing in recent years, see for instance [2,3,6,8,11,15–20,23].

It can be seen that if $f(x_1, x_2)$ is a birational map, then the sequence of the degrees of F^n satisfies a homogeneous linear recurrence with constant coefficients (see [13] for instance). This is governed by the characteristic polynomial $\mathcal{X}(x)$ of a certain matrix associated to F. The other information we get from $\mathcal{X}(x)$ is the *dynamical degree*, $\delta(F)$, which is defined as

$$\delta(F) := \lim_{n\to\infty} \left(\deg(F^n)\right)^{\frac{1}{n}}. \tag{1}$$

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