



Dynamical Classification of a Family of Birational Maps of \mathbb{C}^2 via Algebraic Entropy

Sundus Zafar¹ · Anna Cima¹

Received: 6 July 2018 / Accepted: 19 November 2018 / Published online: 7 December 2018
© Springer Nature Switzerland AG 2018

Abstract

This work dynamically classifies a 9-parametric family of birational maps $f : \mathbb{C}^2 \rightarrow \mathbb{C}^2$. From the sequence of the degrees d_n of the iterates of f , we find the dynamical degree $\delta(f)$ of f . We identify when d_n grows periodically, linearly, quadratically or exponentially. The considered family includes the birational maps studied by Bedford and Kim (Mich Math J 54:647–670, 2006) as one of its subfamilies.

Keywords Birational maps · Algebraic entropy · First integrals · Fibrations · Blowing-up · Integrability · Periodicity · Chaos

Mathematics Subject Classification 14E05 · 26C15 · 34K19 · 28D20 · 37C15 · 39A23 · 39A45

1 Introduction

In this work we consider the family of fractional maps $f : \mathbb{C}^2 \rightarrow \mathbb{C}^2$ of the form:

$$f(x, y) = \left(\alpha_0 + \alpha_1 x + \alpha_2 y, \frac{\beta_0 + \beta_1 x + \beta_2 y}{\gamma_0 + \gamma_1 x + \gamma_2 y} \right), \quad (1)$$

where the parameters are complex numbers.

This family of maps can be extended to the projective plane $P\mathbb{C}^2$ by considering the embedding $(x_1, x_2) \in \mathbb{C}^2 \mapsto [1 : x_1 : x_2] \in P\mathbb{C}^2$ into projective space. The induced map $F : P\mathbb{C}^2 \rightarrow P\mathbb{C}^2$ has three components $F_i[x_0 : x_1 : x_2]$, $i = 1, 2, 3$ which are homogeneous polynomials of degree two. For general values of the parameters the

✉ Sundus Zafar
Sundus@mat.uab.cat

Anna Cima
cima@mat.uab.cat

¹ Universitat Autònoma de Barcelona, Barcelona, Spain