## Zero entropy for some birational maps of $\mathbb{C}^{2*}$

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## Abstract

This work deals with a special case of family of birational maps  $f : \mathbb{C}^2 \to \mathbb{C}^2$ dynamically classified in [9]. In this work we study the zero entropy sub families of f. The sequence of degrees  $d_n$  associated to the iterates of f is found to grow periodically, linearly, quadratically or exponentially. Explicit invariant fibrations for zero entropy families and all the integrable and periodic mappings inside the family f are given.

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## 1 Introduction

Consider the family of fractional maps  $f : \mathbb{C}^2 \to \mathbb{C}^2$  of the form:

$$f(x,y) = \left(\alpha_0 + \alpha_1 x + \alpha_2 y, \frac{\beta_0 + \beta_1 x + \beta_2 y}{\gamma_0 + \gamma_1 x + \gamma_2 y}\right), (\gamma_1, \gamma_2) \neq (0,0)$$
(1)

where the parameters  $\alpha_i$ ,  $\beta_i$ ,  $\gamma_i$ ,  $i \in \{0, 1, 2\}$  are complex numbers.

In this work the family of mappings f(x, y) in (1) is required to be birational in general. The values of parameters  $\alpha_i$ ,  $\beta_i$ ,  $\gamma_i$ ,  $i \in \{0, 1, 2\}$  for which f(x, y) is a birational mapping is discussed in Lemma 1 in this article. The study of the dynamics generated by birational mappings in the plane and their classification is a well discussed topic in recent years as can

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