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Smooth linearisation of planar periodic maps

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Abstract

The celebrated Kerékjártó theorem asserts that planar continuous periodic maps can be continuously linearised. We prove that for each $k \in \{1, 2, \ldots, \infty\}$, $\mathcal{C}^k$-planar periodic maps can be $\mathcal{C}^k$-linearised.

1. *Introduction*

A continuous map $F : \mathbb{R}^n \to \mathbb{R}^n$ satisfying $F^m = \text{Id}$ is called *m-periodic*. Here $F^j = F \circ F^{j-1}$ and m is the smallest positive natural number with this property. Usually, 2-periodic maps are called *involutions*. The simplest examples of periodic maps are found in the class of linear maps. An endomorphism $L : \mathbb{R}^n \to \mathbb{R}^n$ is periodic if it is diagonalisable in $\mathbb{C}$ and all eigenvalues are roots of unity. It seems natural to ask if these are the only possible examples, meaning that nonlinear periodic maps are indeed equivalent to linear maps. The answer to this question depends upon the dimension n and also on the type of equivalence under consideration.

To initiate the discussions we consider the notion of equivalence induced by topological conjugacy. A map $F : \mathbb{R}^n \to \mathbb{R}^n$ is said (globally) $\mathcal{C}^0$-*linearisable* if there exists a homeomorphism $\psi : \mathbb{R}^n \to \mathbb{R}^n$, such that $L = \psi \circ F \circ \psi^{-1}$ is a linear map. The couple (L, ψ) is called a *linearisation* of F. Notice that we have emphasised the global nature in the above definition. Although many results in the theory of dynamical systems are concerned with linearization, most of them are of local nature. This is the case for the well-known Hartman–Grossman theorem.

In dimension $n = 1$ it is not hard to prove that all periodic maps are $\mathcal{C}^0$- linearizable with $L(x) = x$ or $L(x) = -x$. A similar result holds for $n = 2$, now L is either the symmetry or a rotation of angle commensurable with 2π.

THEOREM 1·1. *(Kerékjártó theorem) Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be a continuous m-periodic map. Then F is $\mathcal{C}^0$-linearisable.*

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