

On planar polynomial vector fields with elementary first integrals

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Abstract

We show that under rather general conditions a polynomial differential system having an elementary first integral already must admit a Darboux first integral, and we explicitly characterize the vector fields in this class. We also investigate some exceptional cases, i.e. equations admitting an elementary first integral but not a Darboux first integral. In particular we provide a rather detailed discussion of exceptional elementary first integrals built from algebraic functions of prime degree.

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1. Introduction and survey of results

In the present note we discuss a polynomial vector field

$$X = P \frac{\partial}{\partial x} + Q \frac{\partial}{\partial y}, \quad \text{briefly } X = \begin{pmatrix} P \\ Q \end{pmatrix} \quad (1)$$

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