DISCRETE AND CONTINUOUS
DYNAMICAL SYSTEMS SERIES B
Volume **24**, Number **12**, December **2019**

doi:10.3934/dcdsb.2019153

pp. **6541–6552**

CYCLICITY OF (1, 3)-SWITCHING FF TYPE EQUILIBRIA

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(Communicated by Yuan Lou)

Abstract. Hilbert's 16th Problem suggests a concern to the cyclicity of planar polynomial differential systems, but it is known that a key step to the answer is finding the cyclicity of center-focus equilibria of polynomial differential systems (even of order 2 or 3). Correspondingly, the same question for polynomial discontinuous differential systems is also interesting. Recently, it was proved that the cyclicity of (1, 2)-switching FF type equilibria is at least 5. In this paper we prove that the cyclicity of (1, 3)-switching FF type equilibria with homogeneous cubic nonlinearities is at least 3.

1. **Introduction and the main result.**

A differential system of the form

$$\dot{x} = P(x, y), \qquad \dot{y} = Q(x, y), \tag{1}$$

where the dot denotes derivative with respect to an independent variable t, and P and Q are both polynomials in the real variables x and y, is called a *polynomial differential system* on the plane $\mathbb{R}^2$. The maximum of the degrees of P and Q is referred to the *degree* of system (1). Thus, a planar polynomial differential system of degree one is a *linear differential system*, and a planar polynomial differential system of degree two (or three) is called a *quadratic* (or *cubic*) *differential system*.

A periodic orbit of a differential system which is isolated in the set of all periodic orbits of the system is called a *limit cycle*, which is one of main topics in the qualitative theory of differential equations in the plane (see [5, 10, 17, 19]). The rise of limit cycles near an equilibrium caused by the changes of its stability is called *Hopf bifurcation* (see [15]). The *cyclicity* of that equilibrium is the maximum

2010 *Mathematics Subject Classification.* Primary: 34C07, 34C23, 34C37, 34K18.

Key words and phrases. Hopf bifurcation, cyclicity, discontinuous differential system, limit cycle.

The first author is supported by NSFC #11871355. The second author has been partially supported by the Ministerio de Economía, Industria y Competitividad, Agencia Estatal de Investigación grants MTM2016-77278-P (FEDER) and MDM-2014-0445, the Agència de Gestió d'Ajuts Universitaris i de Recerca grant 2017SGR1617, and the H2020 European Research Council grant MSCA-RISE-2017-777911. The third author is supported by NSFC #11726623 and #11771307.

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