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CENTERS OF DISCONTINUOUS PIECEWISE SMOOTH QUASI–HOMOGENEOUS POLYNOMIAL DIFFERENTIAL SYSTEMS

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Abstract. In this paper we investigate the center problem for the discontinuous piecewise smooth quasi–homogeneous but non–homogeneous polynomial differential systems. First, we provide sufficient and necessary conditions for the existence of a center in the discontinuous piecewise smooth quasi–homogeneous polynomial differential systems. Moreover, these centers are global, and the period function of their periodic orbits is monotonic. Second, we characterize the centers of the discontinuous piecewise smooth quasi–homogeneous cubic and quartic polynomial differential systems.

1. Introduction.

When all the orbits of a differential system in $\mathbb{R}^2$ in a punctured neighborhood of an equilibrium p of the system are periodic, we say that the p is a *center* of the differential system. A center p of a differential system is *global* when all the orbits of the system in $\mathbb{R}^2 \setminus \{p\}$ are periodic.

In the qualitative theory of planar smooth differential systems, the center problem is a classical problem, which consists in determining the existence of a center, i.e. give necessary and sufficient conditions in order that an equilibrium of a differential system in the plane $\mathbb{R}^2$ could be a center. The study of the centers goes back to Poincaré [28] and Dulac [8], and in the present days many questions about them remain open.

It is known that there are three kind of centers for the analytic differential systems in $\mathbb{R}^2$. The *linear type centers* or simply *linear centers* are the centers whose linear part has purely imaginary eigenvalues. The *nilpotent centers* are the centers such that the eigenvalues of their linear part are both zeros, but the linear part is not identically zero. Finally, the *degenerate centers* are the centers whose linear part is identically zero. For more details on these three kinds of centers see for instance [21] and the references quoted there.

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