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Limit cycles of a second-order differential equation

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ABSTRACT

We provide an upper bound for the maximum number of limit cycles bifurcating from the periodic solutions of $\ddot{x}+x=0$, when we perturb this system as follows

$$\ddot{x}+\varepsilon(1+\cos^m\theta)Q(x,y)+x=0,$$

where $\varepsilon>0$ is a small parameter, m is an arbitrary non-negative integer, $Q(x,y)$ is a polynomial of degree n and $\theta=\arctan(y/x)$. The main tool used for proving our results is the averaging theory.

1. Introduction

To determine the number of limit cycles of a differential equation is one of the main problems in the qualitative theory of planar differential system. In 1881 Poincaré [1] defined the notion of *limit cycle* of a planar differential system as a periodic orbit isolated in the set of all periodic orbits of the differential system. And he defined the notion of a *center* of a real planar differential system, i.e. of an isolated equilibrium point having a neighborhood filled with periodic orbits. Later on one way to produce limit cycles is by perturbing the periodic orbits of a center, see for instance the papers [2–5] and the references quoted there.

In [6] Mathieu considered the second order differential equation

$$\ddot{x}+b(1+\cos t)x=0, \tag{1}$$

where b is a real constant. It is called Mathieu equation, which is the simplest mathematical model of an excited system depending on a parameter. The more general Ermakov–Pinney equation is the Mathieu–Duffing type equation

$$\ddot{x}+b(1+\cos t)x-x^{\beta}=0, \tag{2}$$

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