ISOCHRONOUS CENTERS OF A LINEAR CENTER PERTURBED BY HOMOGENEOUS POLYNOMIALS*

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ABSTRACT

In this paper we study isochronous centers of two-dimensional autonomus systems in the plane with linear part of center type and non-linear part given by fourth and fifth degree homogeneous polynomials. We first found necessary conditions for such isochronous centers in polar coordinates. Finally we give a proof of the isochronicity of these systems using different methods.

1. Introduction

We consider the system

$$\begin{aligned} \dot{x} &= -y + X_s(x,y) \ , \\ \dot{y} &= x + Y_s(x,y) \ , \end{aligned} \tag{1}$$

where $. = \frac{d}{dt}$, being $X_s(x,y)$ and $Y_s(x,y)$ *homogeneous polynomials of degree* s, with $s \geq 2$.

The integrable cases for quadratic systems, $s = 2$, and cubic homogeneous systems, $s = 3$, have been studied by several authors, namely N.N. Bautin[1], J. Chavarriga[2], W.A. Coppel[5], N.G. Lloyd[6], V.A. Lunkevich[7], D. Schlomiuck[8] and H. Zoladek[9]. Some integrable cases of system (1) when $s = 4, 5$ have been determinated by J. Chavarriga [10] and [11].

A center is *isochronous* if the period of all integral curves in a neighborhood of the origin is constant. The equations are most easily written when the arc length s is the variable

$$\ddot{s} + k^2 s = 0 \ , \tag{2}$$

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