## INTEGRABILITY OF CUBIC SYSTEMS WITH DEGENERATE INFINITY\*

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## Abstract

In this work we study the integrability of two-dimensional autonomous system in the plane with linear part of center type and non-linear part given by cubic polynomials with degenerate infinity. We give a simple characterisation for all integrable cases in polar coordinates.

## 1 Introduction

We consider the system

$$\frac{dx}{dt} = -\lambda y + X_2(x,y) + X_3(x,y), \quad \frac{dy}{dt} = \lambda x + Y_2(x,y) + Y_3(x,y), \quad (1)$$

where  $X_s(x, y)$  and  $Y_s(x, y)$ , s=2,3, are homogeneous polynomials of degree s, satisfying  $xY_3(x, y) - yX_3(x, y) \equiv 0$  whenever  $X_3^2 + Y_3^2 \not\equiv 0$ . Such systems are a particular class of the cubic systems with degenerate infinity. This name is due to the fact that, in Poincaré compactification of (1), all the infinity is filled up of critical points, see Sotomayor [12]. This problem has been studied by Chen Guang-Qing and Liang Zhao-Jun [4] and recently by Gasull and Prohens [6], in this work he gives an affine classification of quadratic systems with degenerate infinity.

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