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J. CHAVARRIGA (Lleida)

A CLASS OF INTEGRABLE POLYNOMIAL VECTOR FIELDS

Abstract. We study the integrability of two-dimensional autonomous systems in the plane of the form $\dot{x} = -y + X_s(x, y)$, $\dot{y} = x + Y_s(x, y)$, where $X_s(x, y)$ and $Y_s(x, y)$ are homogeneous polynomials of degree s with $s \geq 2$. First, we give a method for finding polynomial particular solutions and next we characterize a class of integrable systems which have a null divergence factor given by a quadratic polynomial in the variable $(x^2 + y^2)^{s/2-1}$ with coefficients being functions of $\tan^{-1}(y/x)$.

1. Introduction. We consider two-dimensional autonomous systems of differential equations of the form

$$\dot{x} = -y + X_s(x, y), \qquad \dot{y} = x + Y_s(x, y), \tag{1.1}$$

where

$$X_s(x, y) = \sum_{k=0}^{s} a_k x^k y^{s-k}, \qquad Y_s(x, y) = \sum_{k=0}^{s} b_k x^k y^{s-k}$$

are homogeneous polynomials of degree s, with $s \geq 2$, and with a_k and b_k, $k = 0, 1, \ldots, s$, being arbitrary real coefficients. Recently, these systems have been studied by several authors (see for instance [1], [3], [5], [6] and [8]), especially in order to obtain information about the number of small amplitude limit cycles and to determine the cyclicity of the origin (see for instance [4] and [7]). In this paper we study their integrability.

Our aim is to find solutions $W(x, y) = 0$ of system (1.1), where $W(x, y)$ is a null divergence factor (this notion will be defined below). In Theorem 1 we give an explicit method for obtaining such a factor, which is used in Theorem 2 to construct a particular class of integrable fields.

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