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# INTEGRABLE SYSTEMS IN THE PLANE WITH CENTER TYPE LINEAR PART

*Abstract.* We study integrability of two-dimensional autonomous systems in the plane with center type linear part. For quadratic and homogeneous cubic systems we give a simple characterization for integrable cases, and we find explicitly all first integrals for these cases. Finally, two large integrable system classes are determined in the most general nonhomogeneous cases.

**1. Introduction.** We consider two-dimensional autonomous systems of differential equations of the form

$$\dot{x} = -y + X(x,y), \quad \dot{y} = x + Y(x,y)\,, \tag{1.1}$$

where $X(x,y)$ and $Y(x,y)$ are analytic functions without linear terms defined in a certain neighbourhood of the origin. In the local study of these systems we find three problems closely related to one another: the determination of the origin's stability, existence and number of local limit cycles around the origin and the determination of first integrals when they exist. Poincaré developed an important technique for the general solution of those problems: it consists in finding a formal power series of the form

$$H(x,y) = \sum_{n=2}^{\infty} H_n(x,y)\,, \tag{1.2}$$

where $H_2(x,y) = (x^2+y^2)/2$ and $H_n(x,y)$ are homogeneous polynomials of degree $n$, so that

$$\dot{H} = \sum_{k=2}^{\infty} V_{2k}(x^2+y^2)^k\,, \tag{1.3}$$

where $V_{2k}$ are real numbers called *Lyapunov constants*. The determination of these constants allows the solution of the three mentioned problems.

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