ALGEBRAIC LIMIT CYCLES IN PIECEWISE LINEAR DIFFERENTIAL SYSTEMS

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Abstract. This paper is devoted to study the algebraic limit cycles of planar piecewise linear differential systems. In particular we present examples exhibiting two explicit hyperbolic algebraic limit cycles, as well as some 1-parameter families with a saddle-node bifurcation of algebraic limit cycles. We also show that all degrees for algebraic limit cycles are allowed.

1. Introduction

The study of algebraic limit cycles of planar polynomial differential equations has a long history. To the best of our knowledge they started to be considered, particularly for quadratic systems, around the late fifty's of last Century by the Chinese and Russian schools, see the survey paper [14] and the references therein. The main motivation of these works was to advance in the knowledge and role of the algebraic solutions of polynomial differential equations and constitutes, somehow, a continuation of the classical works of Darboux and Poincaré in this direction.

For instance, up to now it is known that there are quadratic systems having algebraic limit cycles of degrees 2, 4, 5 or 6, see [4, 16, 17] and their references. Notice that in the smooth world, these type of differential equations are the simplest ones that can have this type of solutions.

In this paper we answer some natural questions about the parallel problem of algebraic periodic solutions but in simple piecewise systems. We consider two linear systems with a straight line of separation and we study the existence, number and degree of the "algebraic" limit cycles in this setting.

In fact, in non-smooth dynamics the differential equations appearing in the simplest models are linear and of the type described above. These models have attracted the attention of many scientists not only because of its simplicity, but for the accuracy of the obtained results comparing with the real observations, see more details in [1, 2, 12].

In order to state precisely our results we introduce first some notations and definitions. Consider the piecewise differential system

$$X^{\pm} \colon (x', y') = (f^{\pm}(x, y), g^{\pm}(x, y)),$$

defined in $\Sigma^{\pm} = \{(x, y) \in \mathbb{R}^2 : \pm x \geq 0\}$. If an equilibrium point of $X^{\pm}$ is in $\Sigma^{\pm}$ it is said that it is a *visible* equilibrium point for the piecewise system. Otherwise it is called an *invisible* equilibrium point.

Notice that over the line $\Sigma = \{(0, y) \in \mathbb{R}^2\}$ the vector field is bivaluated. We say that a point $q \in \Sigma$ is of *crossing* type if both vectors $X^{\pm}(q)$ point inn Σ^+ or Σ^-. Moreover, on our separation line, if $X^+(q)$ or $X^-(q)$ are vertical, then q is a *tangent* point. Otherwise q is of *sliding* type and these points define the *sliding set*. Periodic orbits that have neither sliding part nor tangent points are called *crossing periodic orbits*, otherwise are called

2010 *Mathematics Subject Classification.* 34A05, 34C05, 34C25, 37G15, 37C27.

Key words and phrases. Non-smooth differential system, piecewise linear system, algebraic limit cycle, hyperbolic and double limit cycle, saddle-node bifurcation.