Electronic Journal of Differential Equations, Vol. 2019 (2019), No. 15, pp. 1–25.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu

PHASE PORTRAITS OF QUADRATIC POLYNOMIAL DIFFERENTIAL SYSTEMS HAVING AS SOLUTION SOME CLASSICAL PLANAR ALGEBRAIC CURVES OF DEGREE 4

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Abstract. We classify the phase portraits of quadratic polynomial differential systems having some relevant classic quartic algebraic curves as invariant algebraic curves, i.e. these curves are formed by orbits of the quadratic polynomial differential system.

More precisely, we realize 16 different well-known algebraic curves of degree 4 as invariant curves inside the quadratic polynomial differential systems. These realizations produce 31 topologically different phase portraits in the Poincaré disc for such quadratic polynomial differential systems.

1. Introduction and statement of main results

We call quadratic differential systems, simply *quadratic systems* or (QS), the differential systems of the form

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \tag{1.1}$$

where P and Q are real polynomials in the variables x and y, such that the $\max\{\deg(P), \deg(Q)\} = 2$. Here the dot denotes, as usual, differentiation with respect to the time t. To such a system one can always associate the quadratic vector field $\mathcal{X} = P(x, y)\partial/\partial x + Q(x, y)\partial/\partial y$.

If system (1.1) has an algebraic trajectory curve, which is defined by a zero set of a polynomial, $h(x, y) = 0$. Then it is clear that the derivative of h with respect to the time will not change along the curve $h = 0$, and by the Hilbert's Nullstellensatz (see for instance [5]) we have

$$\frac{dh}{dt} = \frac{\partial h}{\partial x}P + \frac{\partial h}{\partial y}Q = hk, \tag{1.2}$$

where k is a polynomial in x and y of degree at most 1, called the *cofactor* of the *invariant algebraic curve* $h(x, y) = 0$. For more details on the invariant algebraic curves of a polynomial differential system see Chapter 8 of [4].

Recently the quadratic systems have been intensely studied using algebraic, geometric, analytic and numerical tools. More than one thousand papers on these systems have been published, see for instance the books of Ye Yanquian et al. [11], Reyn [10], and Artés et al. [2] and the references quoted therein.

2010 *Mathematics Subject Classification.* 34C15, 34C25.
Key words and phrases. Quadratic differential system; Poincaré disc; invariant algebraic curve.

Submitted June 15, 2018. Published January 28, 2019.