Journal of Nonlinear Modeling and Analysis Website:http://jnma.ca
Volume 1, Number 1, March 2019 pp. 11–26

PERIODIC SOLUTIONS OF THE DUFFING DIFFERENTIAL EQUATION REVISITED VIA THE AVERAGING THEORY*

Rebiha Benterki [1] and Jaume Llibre[2,†]

Abstract We use three different results of the averaging theory of first order for studying the existence of new periodic solutions in the two Duffing differential equations $\ddot{y} + a \sin y = b \sin t$ and $\ddot{y} + ay - cy^3 = b \sin t$, where a, b and c are real parameters.

Keywords Periodic solution, averaging method, Duffing differential equation, bifurcation, stability.

MSC(2010) 34C15, 34C25.

1. Introduction and statement of the main results

Hamel [6] in 1922 gaves the first general results for the existence of periodic solutions of the periodically forced pendulum equation

$$\ddot{y} + a \sin y = b \sin t, \tag{1.1}$$

where the dot denotes derivative with respect to the independent variable t , also called the time, and $y \in \mathbb{S}^1$ is an angle. Four years earlier this equation was the main subject of a monograph published by Duffing [4], who restricted his study to the periodic solutions of the following approximate equation

$$\ddot{y} + ay - cy^3 = b \sin t. \tag{1.2}$$

This equation is now known as the *Duffing differential equation*. The differential equation (1.2) describes the motion of a damped oscillator with a more complicated potential than in the harmonic motion (i.e. when $c = 0$). As usual the parameter a controls the size of stiffness, b controls the amplitude of the periodic driving force, and c controls the amount of nonlinearity in the restoring force. In particular, equation (1.2) models a spring pendulum such that its spring's stiffness only obey approximately the Hooke's law.

Many other different classes of Duffing differential equations have been investigated by several authors. They are mainly interested in the existence of periodic

[†]the corresponding author.
Email address: r.benterki@univ-bba.dz (R. Benterki), jllibre@mat.uab.cat (J. Llibre)
[1] Département de Mathématiques, Université de Bordj Bou Arréridj, Bordj Bou Arréridj 34265, El anasser, Algeria
[2] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain