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Singular values and bounded Siegel disks

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Abstract

Let f be an entire transcendental function of finite order and Δ be a forward invariant bounded Siegel disk for f with rotation number in Herman's class $\mathcal{H}$. We show that if f has two singular values with bounded orbit, then the boundary of Δ contains a critical point. We also give a criterion under which the critical point in question is recurrent. We actually prove a more general theorem with less restrictive hypotheses, from which these results follow.

1. *Introduction*

We consider the dynamical system generated by the iterates of an entire transcendental function $f : \mathbb{C} \to \mathbb{C}$, that is a function which is holomorphic on the complex plane $\mathbb{C}$ and has an essential singularity at infinity (in general, we will omit the word 'transcendental'). In this setup, there is a dynamically natural partition of the phase space into two completely invariant subsets: the *Fatou set* $\mathcal{F}(f)$, formed by those $z \in \mathbb{C}$ for which the family of iterates $\{f^n\}_{n\in\mathbb{N}}$ is normal in the sense of Montel in some neighbourhood of z; and the *Julia set* $\mathcal{J}(f)$, its complement. Orbits in the Julia set exhibit chaotic behavior – in fact, $\mathcal{J}(f)$ is the closure of the repelling periodic points of f.

The Fatou set is open, possibly with infinitely many components, called *Fatou components*. The periodic ones are completely classified into basins of attraction of attracting or parabolic cycles, Siegel disks (topological disks on which a certain iterate of f is conjugate to a rigid irrational rotation of angle θ, called the *rotation number*) or Baker domains (regions on which iterates converge uniformly to infinity). Non-periodic Fatou components are called *preperiodic* if they are eventually mapped to a periodic component, and *wandering* otherwise. Baker and wandering domains are types of Fatou components which appear only in the transcendental setting. For a classification of Fatou components in the transcendental case see e.g. [**Be**].