Nonlinear Dyn
https://doi.org/10.1007/s11071-018-4115-3

ORIGINAL PAPER

Limit cycles bifurcating from a zero–Hopf singularity in arbitrary dimension

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Received: 6 July 2017 / Accepted: 29 January 2018

Abstract For a C^{m+1} differential system on $\mathbb{R}^n$, we study the limit cycles that can bifurcate from a zero–Hopf singularity, i.e., from a singularity with eigenvalues $\pm bi$ and $n-2$ zeros for $n \geq 3$. If the singularity is at the origin and the Taylor expansion of the differential system (without taking into account the linear terms) starts with terms of order m, then ℓ limit cycles can bifurcate from the origin with $\ell \in \{0, 1, \ldots, 2^{n-3}\}$ for $m = 2$ [see Llibre and Zhang (Pac J Math 240:321–341, 2009)], with $\ell \in \{0, 1, \ldots, 3^{n-2}\}$ for $m = 3$, with $\ell \leq 6^{n-2}$ for $m = 4$, and with $\ell \leq 4 \cdot 5^{n-2}$ for $m = 5$. Moreover, $\ell \in \{0, 1, 2\}$ for $m = 4$ and $n = 3$, and $\ell \in \{0, 1, 2, 3, 4, 5\}$ for $m = 5$ and $n = 3$. In particular, the maximum number of limit cycles bifurcating from the zero–Hopf singularity grows up exponentially with n for $m = 2, 3$.

Keywords Limit cycles · Zero–Hopf singularity · Arbitrary dimension

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Mathematics Subject Classification Primary 37J35 · 37K10

1 Introduction and statement of the main results

We recall that a zero–Hopf singularity is an isolated equilibrium point of an n-dimensional autonomous system with $n \geq 3$ with linear part having $n-2$ zero eigenvalues and a pair of purely imaginary eigenvalues. It turns out that its unfolding has a rich dynamics in a neighborhood of the singularity (see for example Guckenheimer and Holmes [4,5], Scheurle and Marsden [12], Kuznetsov [6] and the references therein). Moreover, a zero–Hopf bifurcation may lead to a local birth of “chaos.” More precisely, it was shown that some invariant sets of the unfolding can be obtained from the bifurcation from the singularity under appropriate conditions (cf. [3,12]).

In this paper, we use the first-order averaging theory to study the zero–Hopf bifurcation of C^{m+1} differential systems on $\mathbb{R}^n$ with $n \geq 3$ and $m \leq 5$. We assume that these systems have a singularity at the origin with linear part with eigenvalues $\varepsilon a \pm bi$ and εc_k for $k = 3, \ldots, n$, where ε is a small parameter. Each of these systems can be written in the form

$$\begin{aligned}\dot{x} &= \varepsilon a x - b y \\ &\quad + \sum_{i_1+\cdots+i_n=m} a_{i_1\cdots i_n} x^{i_1} y^{i_2} z_3^{i_3} \cdots z_n^{i_n} + P, \\ \dot{y} &= bx + \varepsilon a y\end{aligned}$$

Published online: 08 February 2018