ESCAPING POINTS IN THE BOUNDARIES OF BAKER DOMAINS*

By

Krzysztof Barański, Núria Fagella,† Xavier Jarque† and Bogusława Karpińska

Abstract. We study the dynamical behaviour of points in the boundaries of simply connected invariant Baker domains U of meromorphic maps f with a finite degree on U. We prove that if $f|_U$ is of hyperbolic or simply parabolic type, then almost every point in the boundary of U, with respect to harmonic measure, escapes to infinity under iteration of f. On the contrary, if $f|_U$ is of doubly parabolic type, then almost every point in the boundary of U, with respect to harmonic measure, escapes to infinity under iteration of f. On the contrary, if $f|_U$ is of doubly parabolic type, then almost every point in the boundary of U, with respect to harmonic measure, has dense forward trajectory in the boundary of U, in particular the set of escaping points in the boundary of U has harmonic measure zero. We also present some extensions of the results to the case when f has infinite degree on U, including the classical Fatou example.

1 Introduction and statement of the results

Let $f : \mathbb{C} \to \widehat{\mathbb{C}}$ be a meromorphic map of degree larger than 1 and consider the dynamical system generated by the iterates $f^n = f \circ \cdots \circ f$. The complex sphere is then divided into two invariant sets: the **Fatou set** $\mathcal{F}(f)$, which is the set of points $z \in \widehat{\mathbb{C}}$, where the family of iterates $\{f^n\}_{n\geq 0}$ is defined and normal in some neighbourhood of z, and its complement, the **Julia set** $\mathcal{J}(f) = \widehat{\mathbb{C}} \setminus \mathcal{F}(f)$, where chaotic dynamics occurs. We refer to [Ber93, CG93, Mil06] for the basic properties of Fatou and Julia sets.

It is well-known that for any polynomial of degree larger than 1, the point at infinity is a super-attracting fixed point and the set of points whose orbits tend to infinity coincides with its immediate basin of attraction. Note that no point in the boundary of this basin tends to infinity under iteration. In the case of a

^{*}Research supported by the Polish NCN grant decision DEC-2012/06/M/ST1/00168.

[†]The second and third authors were partially supported by the Spanish grants MTM2014-52209-C2-2-P, MTM2017-86795-C3-3-P, the Maria de Maeztu Excellence Grant MDM-2014-0445 of the BGSMath and the catalan grant 2017SGR1374.