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On Strictly Convex Central Configurations of the 2*n*-Body Problem

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Abstract
We consider planar central configurations of the Newtonian $2n$-body problem consisting in two twisted regular n-gons of equal masses. We prove the conjecture that for $n \geq 5$ all convex central configurations of two twisted regular n-gons are strictly convex.

Keywords Central configuration · Convex central configuration · n-Body problem · Twisted central configuration

Mathematics Subject Classification 70F10 · 70F15

1 Introduction and Main Result

Consider the planar Newtonian $2n$-body problem of two groups of n bodies in the same plane (x, y) at positions $\mathbf{q}_{ji} \in \mathbb{R}^2, i = 1, \ldots, n, j = 1, 2$, such that all the bodies in the same group have equal mass, m_1 and m_2, respectively. Without lost of generality, we can consider the center of mass at the origin. It is well known (see Saari [5]) that a *central configuration* of the planar $2n$-body problem is a solution $\mathbf{q} = (\mathbf{q}_{11}, \mathbf{q}_{12}, \ldots, \mathbf{q}_{2n}) \in \mathbb{R}^{4n}$ of the equation

$$\nabla U(\mathbf{q}) + w^2 M \mathbf{q} = 0,$$

for some value of w, where U is the Newtonian potential

$$U(\mathbf{q}) = \sum_{j=1}^{2} \sum_{i=1}^{n-1} \sum_{l=i+1}^{n} \frac{m_j^2}{||\mathbf{q}_{ji} - \mathbf{q}_{jl}||} + \sum_{i=1}^{n} \sum_{m=1}^{n} \frac{m_1 m_2}{||\mathbf{q}_{1i} - \mathbf{q}_{2m}||},$$

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