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PHASE PORTRAITS FOR QUADRATIC SYSTEMS HAVING A FOCUS AND ONE ANTISADDLE

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ABSTRACT. We determine all possible phase portraits for quadratic systems having a focus and one antisaddle modulus limit cycles and antisaddle behavior.

1. Introduction. In 1984 Cherkas and Gaiko [**9**] proved that if a quadratic system has a focus (or a center) at the origin and no other finite critical point except an antisaddle (an elementary critical point with index +1), it can be transformed by a linear change of variables into the form

$$\begin{aligned} x' &= \alpha x - y - \alpha x^2 + (a + \alpha\gamma)xy + (b - \gamma + c\alpha)y^2 = P(x,y), \\ y' &= x + \alpha y - x^2 + (\gamma - a\alpha)xy + (\alpha\gamma + c - b\alpha)y^2 = Q(x,y), \end{aligned} \tag{1}$$

where

$$b^2 - 4(a-1)c < 0 \quad \text{and} \quad a > 1. \tag{2}$$

When $\alpha = 0$ and the three focal quantities at $(0,0)$ are zero, the origin is a center. In general, the problem of separating a focus from a center, and once the focus is separated from the center, to discriminate the order and stability of the nonhyperbolic (weak) focus, is not easy. For quadratic systems this problem was solved partially by Kapteyn [**17**] and completely by Bautin [**6**] (see also Li Chengzhi [**19**]), by using the three independent focal quantities associated to a center of a quadratic system.

Also, for suitable values of the parameters, the antisaddle of system (1) can become a center. Since the quadratic systems having a center were classified by Vulpe [**30**] we do not consider them in this paper.

Our main result is the following one.

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