Anais da Academia Brasileira de Ciências (2018) 90(3): 2599-2616
(Annals of the Brazilian Academy of Sciences)
Printed version ISSN 0001-3765 / Online version ISSN 1678-2690
http://dx.doi.org/10.1590/0001-3765201820170829
www.scielo.br/aabc | www.fb.com/aabcjournal

The phase portrait of the Hamiltonian system associated to a Pinchuk map

JOAN CARLES ARTÉS[1], FRANCISCO BRAUN[2] and JAUME LLIBRE[3]

[1,3]Departament de Matemàtiques, Universitat Autònoma de Barcelona, Edifici Cc, Campus de la UAB, Bellaterra, Cerdanyola del Vallès, 08193 Barcelona, Catalonia, Spain
[2]Departamento de Matemática, Universidade Federal de São Carlos, Rodovia Washington Luís, Km 235, Caixa Postal 676, 13565-905 São Carlos, SP, Brazil

Manuscript received on October 22, 2017; accepted for publication on December 13, 2017

ABSTRACT

In this paper we describe the global phase portrait of the Hamiltonian system associated to a Pinchuk map in the Poincaré disc. In particular, we prove that this phase portrait has 15 separatrices, five of them singular points, and 7 canonical regions, six of them of type strip and one annular.

Key words: center, global injectivity, real Jacobian conjecture, Pinchuk map.

INTRODUCTION

As far as we know, the simplest class of non-injective polynomial local diffeomorphisms of $\mathbb{R}^2$ are the Pinchuk maps, constructed by Pinchuk (1994). The existence of these maps disproves the *real Jacobian conjecture*, that a polynomial local diffeomorphism of $\mathbb{R}^2$ is globally injective. One open problem is to know what exactly fails in this conjecture.

One of the most known conditions for a local diffeomorphism to be a global one is that it is proper. The asymptotic variety of a map of $\mathbb{R}^2$ is the set of points where the map is not proper (i.e., points that are limits of the map under sequences tending to infinity). In particular, a local diffeomorphism is a global diffeomorphism if and only if this set is empty. Gwoździewicz (2000) and Campbell (arXiv:math/9812032 in 1998, 2011) calculated the asymptotic variety of two Pinchuk maps in details. Our aim in this paper is to do a similar work, i.e., to describe a Pinchuk map, but now from a different point of view.

Let $U \subset \mathbb{R}^2$ be an open connected set. Let $F = (p,q) : U \subset \mathbb{R}^2 \to \mathbb{R}^2$ be a C^2 local diffeomorphism. Let $H_F(x,y) = (p(x,y)^2 + q(x,y)^2)/2$ and consider the Hamiltonian system

$$\dot{x} = -(H_F)_y(x,y), \quad \dot{y} = (H_F)_x(x,y), \tag{1}$$

where the dot denotes derivative with respect to the time t. The singular points of system (1) are characterized by the following result, that we shall prove below.

Correspondence to: Francisco Braun
E-mail: franciscobraun@dm.ufscar.br