GLOBAL CENTERS IN A CLASS OF QUINTIC POLYNOMIAL DIFFERENTIAL SYSTEM

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ABSTRACT. A center of a differential system in the plane $\mathbb{R}^2$ is an equilibrium point p having a neighborhood U such that $U \setminus \{p\}$ is filled of periodic orbits. A center p is global when $\mathbb{R}^2 \setminus \{p\}$ is filled of periodic orbits. In general is a difficult problem to distinguish the centers from the foci for a given class of differential systems, and also it is difficult to distinguish the global centers inside the centers. The goal of this paper is to classify the centers and the global centers of the following class of quintic polynomial differential systems

$$\dot{x} = y, \quad \dot{y} = -x + a_{05}\, y^5 + a_{14}\, x\, y^4 + a_{23}\, x^2\, y^3 + a_{32}\, x^3\, y^2 + a_{41}\, x^4\, y + a_{50}\, x^5,$$

in the plane $\mathbb{R}^2$.

1. Introduction and statement of the main results

We consider polynomial differential systems

$$(\dot{x}, \dot{y}) = (P(x, y), Q(x, y)), \tag{1}$$

defined in the plane $\mathbb{R}^2$. Here the dot denotes derivative with respect to the time t. We are interested in the subclass of these polynomial differential systems having an equilibrium point whose linear part has eigenvalues purely imaginary. After an affine change of variables and a rescaling of the time (if necessary) such polynomial differential systems can be written into the form

$$(\dot{x}, \dot{y}) = (-y + P_n(x, y), x + Q_n(x, y)), \tag{2}$$

where P_n and Q_n are polynomials of degree n, which do not have neither constant nor linear terms.

The problem of distinguishing whether the equilibrium point at the origin of system (2) is a center or a focus is a classical problem, known as the *center-focus problem*. Even this problem was partially solved by Lyapunov, see [22], it has been studied for some fixed values of the degree n during more than a century by many authors. The only family completely investigated is the family of the polynomial differential systems of degree 2, denoted simply by quadratic

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