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ON THE INTEGRABILITY OF A SPROTT CUBIC CONSERVATIVE JERK SYSTEM

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ABSTRACT. We consider the Sprott cubic conservative jerk differential equation $\dddot{x} - a(1-x^2)x + x^2\dot{x} = 0$ with $a \in \mathbb{R}$. It is known that this differential equation exhibits chaotic motion for some values of the parameter a. Here we study when this differential equation has no chaotic motion, i.e. when it has first integrals, and then we describe its dynamics.

1. Introduction and statement of the main results

J.C. Sprott showed numerically that the following third-order differential equation

$$\dddot{x} - a(1-x^2)x + x^2\dot{x} = 0 \tag{1}$$

is conservative and chaotic for some values of the parameter $a \in \mathbb{R}$, see its differential equation (23) in [11]. Note that this differential equation is a particular jerk differential system, see again [11]. These last years many authors have studied different kinds of jerk differential systems, see for instance [3, 4, 7, 10, 12].

In this paper we study when the differential equation (1) is non-chaotic. More precisely, when this equation is integrable, and in this case we describe its dynamics.

We write the third-order differential equation as the following differential system of first order in $\mathbb{R}^3$

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= z, \\ \dot{z} &= a(1-x^2)x - x^2 y, \end{aligned} \tag{2}$$

with $a \in \mathbb{R}$. We denote by

$$X = X(x,y,z) = y\frac{\partial}{\partial x} + z\frac{\partial}{\partial y} + (a(1-x^2)x - x^2y)\frac{\partial}{\partial z}, \tag{3}$$

the *vector field* associated to the differential system (2).

Since the differential system (2) is invariant under the symmetry

$$\tau(x,y,z) = (-x,-y,-z), \tag{4}$$

its phase portrait is symmetric with respect to the origin of coordinates.

2020 *Mathematics Subject Classification.* 34C05.

Key words and phrases. First integrals, invariant algebraic surfaces, exponential factors, Sprott cubic conservative jerk system.