RESOURCE-CONSUMER DYNAMICS IN DRYLANDS: MODELING THE ROLE OF PLANT-PLANT FACILITATION-COMPETITION SHIFTS WITH A PIECEWISE SYSTEM

LEONARDO PEREIRA COSTA DA CRUZ, JOAN TORREGROSA, MIGUEL BERDUGO, AND JOSEP SARDANYÉS*

Abstract. In drylands, water availability determines plant population densities and whether they cooperate via facilitation or compete. When water scarcity intensifies, plant densities decrease and competition for water surpasses the benefits of soil improvement by facilitator plants, involving an abrupt shift from facilitation to competition. Here, we model this facilitation-competition shift using a piecewise system in a resource species such as grasses studying its impact on a resource-consumer dynamical system. First, the dynamics of each system are introduced separately. The competitive system, by setting conditions to have a monodromic equilibrium in the first quadrant, has no limit cycles. With a monodromy condition in the same quadrant, the cooperative system only has a hyperbolic, small amplitude limit cycle, allowing for an oscillating coexistence. The dynamic properties of the piecewise system become richer. We here prove the extension of the center-focus problem in this particular case, and from a weak focus of order three, we find 3 limit cycles arising from it. We also study the case assuming continuity in the piecewise system. Finally, we present a special and restricted way of obtaining a limit cycle of small amplitude in a pseudo-Hopf bifurcation type. Our results suggest that abrupt density-dependent functional shifts, such as those described in drylands, could introduce novel dynamical phenomena. Our work also provides a theoretical framework to model and investigate sharp density-dependent processes in Ecology.

1. Introduction

Plant-plant interactions are one of the core mechanisms shaping the assemblage of a given community in ecosystems, importantly determining the identity and abundance of each species in a given place [27]. Such interactions can be negative when plants compete for the same resources, but also positive, a process called facilitation [8]. Facilitation is especially important in stressful environments [7], such as drylands (sites where it rains less than 65% of what is evaporated [11]), where plants experience a chronic water deficit. In these systems facilitation emerges because plants, by shading and increasing soil organic content increase soil moisture in their surroundings [17, 26], creating micro-environmental conditions that promote the recruitment and growth of other species [25]. However, recent studies have found that facilitation does not increase when the environment gets drier within drylands: it lessens its importance to drive species occurrence as aridity increases [4, 35]. This occurs due to reasons that are still not clear [32] but probably involve: (i) increasing aridity affects the quantity (system gets less productive) and the quality (as the soil is also less fertile with increasing aridity) of their litter, thus of the soil organic matter that ultimately improves microenvironmental conditions [6]; (ii) increasing difficulty in producing an effective soil amelioration for recruitment due to harsher climatic conditions [35]; (iii) shifts in the plant species in the community as aridity increases [4, 32], emerging

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* Corresponding author.