Dynamics of a Two-Level Laser Model with Delay

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In 1982, Arecchi *et al.* proposed a simple two level laser model to interpret the first evidence of chaos and generalized multistability in a Q-switched CO2 laser. In this framework, laser dynamics is described by means of a set of two ordinary differential equations for the photon number and the population inversion between the two resonant levels. A sinusoidal function accounted for cavity loss modulation. In this work, we first prove the existence of a periodic orbit for the original two level non-autonomous laser. Then, we transform this model into a four-dimensional autonomous dynamical system in order to provide a mathematical analysis which confirms the seminal results already obtained. Finally, by replacing the sinusoidal loss modulation with a delayed function of photon number we confirm the occurrence of chaos and multistability for such a delayed model with delay times of the order of reciprocal of the modulation frequencies.

I. INTRODUCTION

In the beginning of the eighties, a pioneer experiment on chaotic behavior in a CO2 laser with periodic modulation applied to the cavity loss parameter, was presented [1]. A simple two-level laser model was used for modeling the observed dynamics including generalized multistability [5]. The paper is organized as follows. In the next section, after having briefly recalled the set of two differential ordinary differential equations of the two-level laser model of 1982 and its original parameters, we provide its corresponding nonlinear second order non-autonomous ordinary differential equation (jerk form) and also its corresponding three-dimensional autonomous dynamical system. Then, we show that the main dynamics features (phase portraits, bifurcation diagrams) of these two models are perfectly identical to the original one. In Sec. 3, we transform this two-level non-autonomous model into a four-dimensional autonomous dynamical system and we provide a mathematical analysis that confirms the seminal results already obtained by Arecchi *et al.* [1]. In Sec. 5, by replacing the sinusoidal loss modulation by a delayed variable, we show that such two-level laser model with delay is perfectly analogous to the one analyzed by Grigorieva *et al.* [3] and for which they demonstrated multistability in a local vicinity of the stationary state.

II. TWO-LEVEL NON-AUTONOMOUS LASER MODEL

A. Original model

According to Arecchi *et al.* [1], the laser dynamics is modeled with a set of two differential equations for the photon number n and the population inversion Δ between the two resonant levels as follows (see Meucci and Ginoux [4] for more details):

$$\begin{aligned}\frac{dn}{dt} &= -k_1\left[1 + m\cos(\Omega t)\right] n + Gn\Delta,\\ \frac{d\Delta}{dt} &= -\gamma\Delta + R - 2Gn\Delta,\end{aligned} \tag{1}$$

where k_1 is the decay rate of photon number, γ is the population inversion decay rate and R is the pump rate, G is the field-matter coupling constant and $m\cos(\Omega t)$ is a sinusoidal signal with amplitude m and modulation frequency $f_{mod} = \Omega/2\pi$ applied to the optical cavity.

B. Rescaled form

Introducing the normalized photon number $x = n/(\gamma/2G)$, the normalized population inversion $y = \Delta/(k_1/G)$ and the non-dimensional time $t' = \gamma t$ the dynamical system becomes