GLOBAL ASYMPTOTIC STABILITY IN QUADRATIC SYSTEMS

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Abstract. A classical problem in the qualitative theory of differential systems that is relevant for its applications, is to characterize the differential systems which are globally asympotically stable, that is differential systems having a unique equilibrium point for which all their orbits, with the exception of the equilibrium point, tend in forward time to this equilibrium point.

Here we provide four conditions that characterize the global asymptotic stability for planar polynomial differential systems. Using these four conditions we characterize all planar quadratic polynomial differential systems that are globally asymptotically stable.

1. Introduction and statement of the main results

Let $f = (f_1, f_2)\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a polynomial map. In this paper we deal with the *polynomial differential systems*

$$\dot{x} = f_1(x, y), \quad \dot{y} = f_2(x, y), \tag{1}$$

where the dot denotes derivative with respect to the time t.

These systems are *globally asymptotically stable* if they have an equilibrium point p such that any orbit $(x(t), y(t))$ with maximal interval (α, ω) different from p satisfies that $x(t)^2 + y(t)^2 \to p$ as $t \to \omega$.

To find conditions which guarantee global asymptotic stability of an equilibrium point in a planar polynomial differential system is a difficult problem. Lyapunov's function method is probably the most common method used, but in general to find a Lyapunov function is not easy.

There is a result proven in 1993 and known as the Markus-Yamabe conjecture which provides sufficient but not necessary conditions for the global asymptotic stability in the plane. These conditions are that the differential system has a unique equilibrium point, the trace of the Jacobian matrix Df is negative and its determinant is positive for all $(x, y) \in \mathbb{R}^2$. While the Markus-Yamabe conditions for C^1 differential systems in $\mathbb{R}^2$ guarantee asymptotic stability in $\mathbb{R}^2$ (see [8, 10, 11]) this is not the case in $\mathbb{R}^n$ with $n > 2$, see [2, 4].

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