THE CLASSICAL AND IMPROVED EULER-JACOBI FORMULA AND POLYNOMIAL VECTOR FIELDS IN $\mathbb{R}^n$

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Abstract. The classical and the new Euler-Jacobi formulae for simple and double points provide an algebraic relation between the singular points of a polynomial vector field and their topological indices.

Using these formulae we obtain the geometrical configuration of the singular points together with their topological indices for several classes of polynomial differential systems in $\mathbb{R}^n$ when these differential systems, having the maximum number of singular points, either all their singular points are simple, or at most one singular point is double (i.e. it has multiplicity two).

1. Introduction and statement of the main results

Consider the polynomial differential system in $\mathbb{R}^n$

$$\dot{x}_i = P_i(x_1, \dots, x_n), \quad i = 1, \dots, n \quad n \geq 2 \tag{1}$$

where $P_i(x_1, \dots, x_n)$ are real polynomials such that $\deg(P_1) \in \{1, 2\}$, $\deg(P_2) = m$ with $m \in \mathbb{N}$ if $\deg(P_1) = 1$ and $m = 2, 3, 4, 5$ if $\deg(P_1) = 2$, and $\deg(P_i) = 1$ for $i \geq 3$. We assume that system (1) has either m singular points, or $m - 1$ singular points one of these singular points is double if $\deg(P_1) = 1$, and if $\deg(P_1) = 2$ then it has either $2m$ singular points, or $2m - 1$ singular points and one of these singular points is double. In the case in which there are $\deg(P_1)m$ finite singular points we use the classical Euler-Jacobi formula (a proof of the classical Euler-Jacobi formula can be found in [1]), and the case in which there are $\deg(P_1)m - 1$ finite singular points, the classical Euler-Jacobi formula is not valid anymore but Gasull and Torregrosa in [6] provided a generalization of the classical Euler-Jacobi formula in the case that the system has one double point and we will use such a formula. Using these formulae we obtain all the possible distributions of the singular points of system (1) when it has either $\deg(P_1)m$, or $\deg(P_1)m - 1$ singular points with $m \in \mathbb{N}$ when either $\deg(P_1) = 1$, or $m = 2, 3, 4, 5$ and $\deg(P_1) = 2$.

Note that assuming that there are finitely many singular points all the singular points lie in the plane Π not necessarily invariant given by the intersection of the $n - 2$ hperplanes $P_i(x_1, \dots, x_n) = 0$ for $i = 3, \dots, n$. On that plane we can reduce system (1) to the planar polynomial differential system

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y) \tag{2}$$

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