ON THE GRAPHICS IN QUADRATIC DIFFERENTIAL SYSTEMS

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Abstract. We denote by **QS** the whole class of planar quadratic differential systems. In this article we are interested in the graphics of this family. A perturbation within **QS** of a graphic in a system of this family may produce limit cycles. Hence graphics play an important role in Hilbert's 16th problem for **QS**. In 1994 a program to prove the existential part (or the finiteness part) of Hilbert's 16th problem for **QS** was devised by Dumortier, Roussarie and Rousseau. More precisely they reduced this problem to proving the finite cyclicity for only 121 graphics occurring in **QS**. Our article has two goals: (i) Our first goal is to define for the first time equivalence relations for graphics occurring in **QS**, among them one using the here defined notion of geometric configuration of singularities of a graphic in **QS**. (ii) Our second goal is to do a rather in depth study of the paper in which the program is presented and to inform the reader about the progress on this program. We also inform the reader about literature on global studies of families of quadratic differential systems where graphics can be viewed appearing in their natural context, along with their perturbations producing limit cycles in bifurcation diagrams. .

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1. Introduction and basic notions

A real planar polynomial differential system is a differential system of the form

$$\frac{dx}{dt} = p(x, y), \qquad \frac{dy}{dt} = q(x, y), \tag{1}$$

where $p(x, y), q(x, y)$ are polynomials in x, y with real coefficients ($p, q \in \mathbb{R}[x, y]$). We call *degree* of such a system the number $\max(\deg(p), \deg(q))$. We alternatively use also the polynomial vector field

$$X = p(x, y)\frac{\partial}{\partial x} + q(x, y)\frac{\partial}{\partial y},$$

associated to the differential system (1).

A quadratic differential system (or simply quadratic system) is a polynomial differential system of degree 2. The parameter space of the quadratic systems is $\mathbb{R}^{12}$ the space formed by all their coefficients. We denote the class of all quadratic differential systems by **QS**.

Planar polynomial differential systems occur very often in various branches of applied mathematics and they also have theoretical importance. Several problems on polynomial differential systems, which were stated more than one hundred years ago, are still open: the second part of Hilbert's 16th problem stated by Hilbert in 1900, the problem of algebraic integrability stated by Poincaré in 1891 [50], the problem of the center stated by Poincaré in 1885 [48], and problems on integrability resulting from the work of Darboux [21] published in 1878. With the exception of the problem of the center, which was solved only for quadratic differential