LIMIT CYCLES OF DISCONTINUOUS PIECEWISE DIFFERENTIAL SYSTEMS IN $\mathbb{R}^2$ SEPARATED BY ONE OR TWO PARALLEL STRAIGHT LINES AND FORMED BY ARBITRARY LINEAR HAMILTONIAN SYSTEMS

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Abstract. In this paper we deal with discontinuous piecewise linear differential in the plane, separated by one or two parallel straight lines and formed by arbitrary linear Hamiltonian systems. We prove that such systems separated by one straight line have no limit cycles and the ones separated by two straight lines can have at most one limit cycle and it is reached. Our results contain as particular cases the results of four previous papers.

1. Introduction and Statement of the Main Results

Limit cycles of planar differential systems were defined by Poincaré [20]. The study of them plays an important role for understanding the dynamics of many differential systems, see for example [2, 3, 7, 12, 13, 14, 23, 24]. After these works, the non-existence, existence, uniqueness and other properties of limit cycles were studied extensively by mathematicians and physicists, and more recently also by chemists, biologists, economists, etc, see for instance the books [8, 26].

The study of discontinuous differential systems has been developed at a very fast pace in recent years and it becomes a common relationship between mathematics, physics and engineering. They modelling phenomena in mechanical systems and electronic circuits, for instance dry friction or switches, see [4, 5, 6]. Many authors have contributed to the study of discontinuous differential systems. Look at references [1, 10, 15, 19, 22, 25] and the ones quoted in [25].

However there exist many problems that are modeled by using *discontinuous piecewise linear differential systems* separated by straight lines, see for instance [14, 16]. The Filippov's convention [10] allows to define two kind of limit cycles of discontinuous piecewise linear differential systems, the so called *sliding limit cycles* and the *crossing limit cycles*. Sliding limit cycles contain *sliding points* on the line of discontinuity and *crossing limit cycles* contain only *crossing points*. Here we only work with crossing limit cycles, or simply *limit cycles*.

A Hamiltonian system in $\mathbb{R}^2$ is of the kind

$$\dot{x} = -H_y(x,y), \quad \dot{y} = H_x(x,y), \tag{1}$$

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