PERIODIC ORBITS NEAR AN EQUILIBRIUM FOR HAMILTONIAN SYSTEMS WITH 2 DEGREES OF FREEDOM

JAUME LLIBRE

ABSTRACT. We provide explicit expressions for the initial conditions of the families of periodic orbits bifurcating from the equilibrium point localized at the origin of coordinates when the eigenvalues of the linear part at the origin of coordinates of the Hamiltonian system with Hamiltonian

$$H(q_1,q_2,p_1,p_2)=\frac{1}{2}(p_1^2+p_2^2)+V(q_1,q_2)$$

are purely imaginary, being $V(q_1,q_2)$ an arbitrary C^4 function in a neighborhood of the origin.

1. Introduction and statement of the main result

The study of the existence of periodic orbits near an equilibrium point of a Hamiltonian system has a long history see, for instance, the theorems of Lyapunov [2], Weinstein [5, 6], Moser [3], Barreira et al. [1], and the references cited therein. These theorems provide sufficient conditions for the existence of periodic orbits emanating from an equilibrium point of a differential system with a first integral.

The objective of this paper is to study the periodic orbits bifurcating from the equilibrium point localized at the origin of coordinates when the eigenvalues of the linear part at the origin of coordinates of the Hamiltonian system with Hamiltonian

$$H(q_1,q_2,p_1,p_2)=\frac{1}{2}(p_1^2+p_2^2)+V(q_1,q_2) \tag{1}$$

are purely imaginary, being

$$V(q_1,q_2)=\sum_{i+j=2}^{3} a_{ij}q_1^i q_2^j+\text{Lagrange's error},$$

an arbitrary C^4 function in a neighborhood of the origin.

We note that the mentioned results of Lyapunov, Weinstein, Moser and Barreira et al. cannot be applied for studying the periodic orbits bifurcating from the equilibrium point $(0,0,0,0)$ of the Hamiltonian system defined by the Hamiltoninan (1).

Our main result is the following one.

Theorem 1. *Consider the Hamiltonian system defined by the Hamiltonian* (1). *Then from the equilibrium point* $(0,0,0,0)$ *of this Hamiltonian system bifurcate four families of periodic orbits parametrized by the values* h *of the Hamiltonian* (1) *starting at* $h=0$ *under*

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