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SOLUTION OF THE CENTER PROBLEM FOR A CLASS POLYNOMIAL DIFFERENTIAL SYSTEMS

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ABSTRACT. Consider the polynomial differential system of degree m of the form

$$\begin{aligned}\dot{x} = &\quad -y(1+\mu(a_2x-a_1y)) + x(\nu\,(a_1x+a_2y)+\Omega_{m-1}(x,y)),\\ \dot{y} = &\quad x(1+\mu(a_2x-a_1y)) + y(\nu\,(a_1x+a_2y)+\Omega_{m-1}(x,y)),\end{aligned}$$

where μ and ν are real numbers such that $(\mu^2+\nu^2)(\mu+\nu(m-2))(a_1^2+a_2^2)\neq 0$, $m>2$ and $\Omega_{m-1}(x,y)$ is a homogenous polynomial of degree $m-1$. A conjecture, stated in [9] suggests that when $\nu=1$, this differential system has a weak center at the origin if and only if after a convenient linear change of variable $(x,y)\longrightarrow(X,Y)$ the system is invariant under the transformation $(X,Y,t)\longrightarrow(-X,Y,-t)$. For every degree m we prove the extension of this conjecture to any value of ν except for a finite set of values of μ.

1. Introduction and statement of the main results

Let

$$\mathcal{X} = P\frac{\partial}{\partial x} + Q\frac{\partial}{\partial y},$$

be the real planar polynomial vector field associated to the real planar polynomial differential system

$$\dot{x} = P(x,y), \quad \dot{y} = Q(x,y). \tag{1}$$

As usual the dot denotes derivate with respect to the time t. In what follows we assume that the origin $O := (0,0)$ is a singular point, i.e. $P(0,0) = Q(0,0) = 0$.

The singular point O is a *center* if there exists an open neighborhood U of O where all the orbits contained in $U \setminus \{O\}$ are periodic.

The study of the centers of the polynomial differential systems (1) has a long history. The first works are due to Dulac [5] and Poincaré [14]. Later on where developed by Liapunov [11], Bendixson [3], Frommer [6] and many others.

In this paper we shall study the polynomial vector field

$$\mathcal{X} = (-y+X)\frac{\partial}{\partial x} + (x+Y)\frac{\partial}{\partial y} \tag{2}$$

associated to the real planar polynomial linear type center

$$\dot{x} = -y + X, \quad \dot{y} = x + Y, \tag{3}$$

For X and Y polynomials starting with terms of second order the *center-focus problem* asks about the conditions on the coefficients of X and Y under which the origin of system (3) is a center.

A first mechanism for solving the focus-center problem is the Poincaré-Liapunov Theorem.

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