HOPF BIFURCATION FOR A CLASS OF PREDATOR-PREY SYSTEM WITH SMALL IMMIGRATION

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Abstract. The subject of this paper concerns with the bifurcation of limit cycles for a predator-pray model with small immigration. Since, in general, the biological systems are not isolated, taking into account immigration in the model becomes it more realistic. In this context, we deal with a model with a Holling type I function response and study, using averaring theory of second order, the Hopf bifurcation that can emerge under small perturbation of the biological parameters.

1. Introduction

The predator-prey models describe the dynamics of populations in a predator-prey relationship. This kind of models have become of great interest, specially for describing and solving many problems in biology, ecology, medicine, among others. See for instance, [9, 8, 1, 10].

One special question in this area is related with the coexistence or not of the species. In this context the presence of non trivial equilibrium points or limit cycles play an important role. A limit cycle typically occurs when the interactions between predators and prey lead to cyclic behaviour in their population sizes. Many works in predator-prey differential systems have demonstrated that its dynimics can exhibit either cyclic oscillation or divergente extinction of one species.

In this context the presence of immigrants in the species is of biological interest because, in general, in nature most systems are not isolated.

Several authors have analyzed the effects of the presence of immigrants in one or more species. For example, in [2, 3, 4, 18] the authors have used delay equations because delayed migration can occur when the individuals encounter some barriers. Also, in [15] the authors analyze the asymptotic stability in different predator-pray models with small immigration, and in [14] they study a two-dimensional problem that consider immigration in both species.

Also, in higher dimension, [13] studies the existence and stability of equilibrium points and Hopf bifurcation in a three dimensional predator-prey with constant immigration rate in the predator, the prey, and the competitor of the prey species.

Key words and phrases. hyperchaotic system, periodic orbit, limit cycle, Hopf bifurcation, averaging equation.