

# CONTINUOUS-DISCONTINUOUS PIECEWISE DIFFERENTIAL SYSTEMS WITH TWO PIECES SEPARATED BY A NON-REGULAR LINE

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ABSTRACT. During the first years of this century many papers have been published on the piecewise differential systems. The interest on these systems mainly is due to their increasing number of applications for modelling many natural phenomena. One of the main difficulties for understanding the dynamics of these systems consists in controlling their limit cycles. So there are many papers studying the limit cycles of continuous and discontinuous piecewise differential systems, but as far as we know this is the first paper studying the limit cycles of a class of piecewise differential systems such that in a part of the line of separation between the two differential systems forming the piecewise differential system is continuous and in the other part it is discontinuous.

## 1. INTRODUCTION AND RESULTS

In the 1920's started the study of the piecewise differential systems with the works of Andronov, Vitt and Khaikin, see the book [2]. Now such differential systems continue receiving the attention of many researchers due to the fact that they model processes appearing in mechanics, electronics, economy, etc., see for more details the books of di Bernardo et al. [3] and Simpson [15], the survey of Makarenkov and Lamb [14], and the hundreds of references cited in them.

The easiest continuous piecewise differential systems are the ones having only two pieces separated by a straight line in the plane  $\mathbb{R}^2$  and formed by two linear differential systems. Lum and Chua in 1990 conjectured in [12, 13] that such piecewise differential systems have at most one limit cycle. We recall that a limit cycle is an isolated periodic orbit in the set of all periodic orbits of a differential system. The previous conjecture was proved in 1990 by Freire et al. [8]. Later on a distinct and shorter proof was given in 2013 by Llibre, Ordóñez and E. Ponce [11], and more recently in 2021 a new proof has been given by Carmona, Fernández-Sánchez and Novaes [5].

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