ON THE LIMIT CYCLES OF THE DIFFERENTIAL EQUATIONS $dr/d\theta = a_n(\theta)r^n + a_{n-1}(\theta)r^{n-1} + \cdots + a_0(\theta)$

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ABSTRACT. One of the more studied differential equations is $dr/d\theta = a_n(\theta)r^n + a_{n-1}(\theta)r^{n-1} + \cdots + a_0(\theta)$ defined on the cylinder $(\theta, r) \in \mathbb{S}^1 \times \mathbb{R}$. The main difficulty for describing the dynamics of these differential equations is to control their number of limit cycles. Recall that a limit cycle is a periodic orbit isolated in the set of all periodic orbits.

There are many papers studying the limit cycles of such differential equations. The objective of this paper is to summarize the known results on the limit cycles of these differential equations which now appear scattered in many papers.

1. LINEAR, RICCATI AND ABEL DIFFERENTIAL EQUATIONS

Consider the following differential equations on the cylinder $(\theta, r) \in \mathbb{S}^1 \times \mathbb{R}$

$$\frac{dr}{d\theta} = a_n(\theta)r^n + a_{n-1}(\theta)r^{n-1} + \cdots + a_0(\theta), \tag{1}$$

where the functions $a_i(\theta), i = 0, \cdots n$, are continuous and 2π-periodic. If $r(\theta, r_0)$ is the solution of equations (1) such that $r(0, r_0) = r_0$ satisfying that $r(2\pi, r_0) = r_0$, then we say that it is a *periodic solution* of equations (1).

If the periodic solution $r(\theta, r_0)$ is isolated in the set of all periodic solutions of equation (1), then we say that $r(\theta, r_0)$ is a *limit cycle* of equation (1).

Equation (1) with $n = 1$ is a *linear differential equation* having at most one limit cycle, see for instance [15]. While for $n = 2$ it is called a *Riccati equation* with at most two limit cycles, see [23, 25]. For $n = 3$ equation (1) is known as an *Abel equation.* If $a_3(\theta) > 0$ Pliss [27] proved that the Abel equation has at most three limit cycles (see also

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