LIMIT CYCLES FOR A CLASS OF DISCONTINUOUS PIECEWISE DIFFERENTIAL SYSTEMS IN $\mathbb{R}^3$ SEPARATED BY A QUADRIC

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Abstract. It is known that the piecewise differential systems in the plane $\mathbb{R}^2$ which are formed by linear centers and separated by a straight line of discontinuity have no limit cycles, but if they are separated by other types of discontinuity curves, such as conics, then they can have limit cycles. The limit cycles of the piecewise differential systems in $\mathbb{R}^2$ have been studied intensively in the last years but this is not the case for piecewise differential systems in the space $\mathbb{R}^3$.

The goal of this paper is to study the maximum number of limit cycles of discontinuous piecewise differential systems in $\mathbb{R}^3$ separated by a quadric that defines a surface, and formed by linear differential systems similar to linear centers in the plane.

1. Introduction and statement of the main results

A *discontinuous piecewise differential system* or simply a *piecewise differential system* in $\mathbb{R}^3$ is formed by the $n-$tuple $Z = (X_1, ..., X_n)$ of C^r differential systems in $\mathbb{R}^3$ with $r \geq 1$, where n is the number of connected components $\mathcal{R}_i$ of $\mathbb{R}^3 \setminus \Sigma$, and the *discontinuity surface* Σ is defined by $\Sigma = h^{-1}(0)$, where $h : U \subset \mathbb{R}^3 \longrightarrow \mathbb{R}$ is a C^r smooth function, having 0 as a regular value. Hence

$$Z(x, y, z) = X_i(x, y, z), \text{ if } (x, y, z) \in \mathcal{R}_i.$$

In order to establish a definition for the trajectories of Z we must have a criterion for the transition of the trajectories between two distinct components $\mathcal{R}_i$ and $\mathcal{R}_j$ with a common boundary of the discontinuity surface Σ.

The contact between the discontinuity surface Σ and the differential system X_i is described by the directional derivative of h with respect to the vector field X_i, i.e.

$$X_i h(\mathbf{x}) = \langle \nabla h(\mathbf{x}), X_i(\mathbf{x}) \rangle .$$

Here $\langle ., . \rangle$ denotes the usual inner product of the space $\mathbb{R}^3$. Following the Filippov rules introduced in [9], the discontinuity surface Σ is divided into three regions.

The points $\mathbf{x}$ of Σ where both differential systems X_i and X_j simultaneously point outwards (i.e. $X_i h(\mathbf{x}) > 0, X_{i+1}h(\mathbf{x}) < 0$), or inwards (i.e. $X_i h(\mathbf{x}) < 0, X_{i+1}h(\mathbf{x}) > 0$) define the *escaping* Σ^e or *sliding* Σ^s regions, respectively. While the points $\mathbf{x}$ of Σ such that $X_i h(\mathbf{x}) \cdot X_{i+1}h(\mathbf{x}) > 0$, define the *crossing region* Σ^c. The points of Σ which are not in $\Sigma^c \cup \Sigma^e \cup \Sigma^s$ are the *tangency* points of X_i or of X_j with Σ.

A periodic orbit of a differential system which is isolated in the set of all periodic orbits of the system is called *limit cycle*. In the qualitative theory of the dynamical systems the analysis of the existence and the maximum number of limit cycles in a differential system play an important role for understanding the dynamics of the differential system. On the other hand the limits cycles model many natural phenomena, see for instance [2, 17, 26].

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