LIMIT CYCLES OF A CLASS OF DISCONTINUOUS PIECEWISE DIFFERENTIAL SYSTEMS

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1. Introduction and Statement of the Main Results

A limit cycle is a periodic orbit of a differential system which is isolated in the set of all periodic orbits of the system. Limit cycles are one of the most fundamental objects of the differential systems in the plane $\mathbb{R}^2$, i.e. in order to determine the dynamics of a given planar differential system we need to know if it has or not limit cycles, and if it has how many, and which are their distribution in the plane.

Limit cycles appear in many areas of research. For example, for explaining physical phenomena like in van der Pol equation [17], or in the Belousov-Zhavotinsky model [3], ... In addition, the limit cycles are useful in control theory, electrical circuits, economics, etc.

The study of the piecewise differential systems began with Andronov, Vitt and Khaikin, and untill nowadays these systems have received attention of many researchers, mainly due to their applications for modeling many natural phenomena that appear in electronics, mechanics, economy, etc., see for example, the books by di Bernardo *et al.* [2008] and Simpson [2010], the survey of Makarenkov and Lamb [2012], as well as the lots of references cited in these last three works.

The simplest class of discontinuous piecewise differential systems is the planar one, formed by two pieces separated by a straight line having a linear differential system in each piece. In this specific class, several authors have tried to determine the maximum number of limit cycles in this type of discontinuous piecewise differential system, but it remains open to know if this maximum is three, see for instance [12] and the references inside.

Here we deal with the discontinuous piecewise differential systems of the form

$$(1)\qquad (\dot{x},\dot{y}) = \mathbf{X}(x,y) = \begin{cases} \mathbf{X_1}(x,y) = (f_1(x,y), g_1(x,y)), & \text{if} \quad (x,y)\in R_1,\\ \mathbf{X_2}(x,y) = (f_2(x,y), g_2(x,y)), & \text{if} \quad (x,y)\in R_2,\end{cases}$$

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